

TOPPERS VILLAGE



Class 11 Mathematics

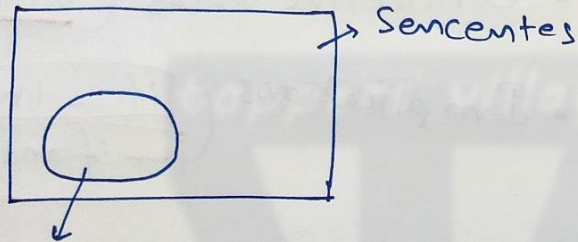
Chapter 14: Mathematical Reasoning

Class Notes (pdf)

YouTube पर इस Chapter का FREE Video देखिए ... (Video की Link नीचे है) ↓↓↓
<https://youtu.be/3NHwRL8jlgs>

Mathematical Reasoning

→ JEE Main **
→ Class - 11

Statements (P)

(~~the~~ those sentences which can be recognised as True (T) or False (F) but not both.)

e.g. $P: 2+2=5$ (F) ✓

$Q: \text{Humans have 7 eyes.}$ (F) ✓

$R: \text{Sun rises in the east.}$ (T) ✓

$S: \text{She is smart.}$ X

$Y: \text{You should not tell a lie.}$ X

Statement $\rightarrow$ T/F
(P)

Truth values

Truth table.

P
T
F

P	Q
T	T
T	F
F	T
F	F

Negation of a statement :

not P

(P)

($\sim P$)

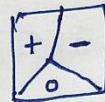
(to change the truth value of a statement)

"it is not the case that"

"it is wrong that"

P : 2 is negative number. (F)

✓ $\sim P$: 2 is not negative number. (T)



$\sim P$: 2 is positive number $\times$ not negation

Q : 2 multiplied by 2 is not 5. (T)

$\sim Q$: 2 multiplied by 2 is 5. (F)

Truth table,

P	$\sim P$
T	F
F	T

$$\sim(\sim P) = P$$

Compound Statements:

(2 or more statements connected with some words)

P : It rains today.

✓ Q : I go to play.

"It rains today and I go to play."

- Quantifiers (there exists, for every)

- Connective words (and, or)

- Implications (if — then —
— if and only if)

① And ($\wedge$): (p and $q \equiv p \wedge q$)

p : Ram needs Voter ID for form filling.

q : Ram needs Driving licence for form filling.

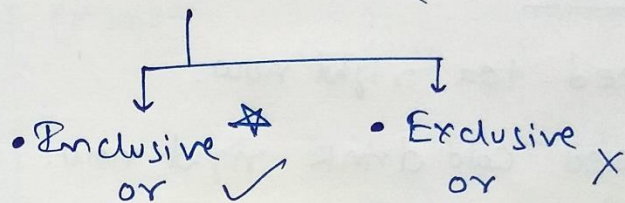
$p \wedge q$: Ram needs Voter ID and Driving licence for form filling.

Truth table.

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$$p \wedge q \equiv q \wedge p$$

② or ($\vee$): (p or $q \equiv p \vee q$)



• Inclusive or: $\star$

p : I need voter ID for form filling

q : I need Driving licence for form filling

$p \vee q$: I need voter ID or DL for form filling.

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

$$\star$$
$$p \vee q \equiv q \vee p$$

• Exclusive or :

p : I need tea right now.

q : I need cold drink right now.

$p \vee q$: I need tea or cold drink right now.
 ↑
 (exclusive)

$\begin{matrix} T & T & \rightarrow & F \\ (p) & (q) & & (p \vee q) \end{matrix}$

★ Note:
 $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$
 $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
 De Morgan's Law ↗

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

LHS
RHS

(II) If _ then _
→
⇒

if p then q $\equiv p \rightarrow q$

p : It rains today.

q : I go to play.

$p \rightarrow q$: If it rains today then I go to play.

Note:

$p \rightarrow q \equiv$ if p then q

$(p \Rightarrow q) \equiv$ p implies q

$\equiv$ p is sufficient for q

$\equiv$ q is necessary for p

$\equiv$ p only if q

$\equiv$ if $(\sim q)$ then $(\sim p)$

truth table if — then —

P	Q	$P \rightarrow Q$	$Q \rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$$\boxed{P \rightarrow Q \neq Q \rightarrow P}$$

④ If and only if ($\leftrightarrow$)

$$(\underline{P} \text{ if and only if } \underline{Q}) \equiv \begin{pmatrix} P \leftrightarrow Q \\ P \leftrightarrow Q \end{pmatrix}$$

P: Ram gets Success.

Q: Ram works hard.

$P \leftrightarrow Q$: Ram gets success if and only if Ram works hard.

$P \leftrightarrow Q \equiv$ 'P' if and only if 'Q'.

$(P \leftrightarrow Q)$

$\equiv$ P iff Q.

$\equiv$ P implies that and implied by Q.

$\equiv$ P is necessary and sufficient condition for Q.

$\equiv$ if P then Q and if Q then P.

P	Q	$P \leftrightarrow Q$	$Q \leftrightarrow P$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

$$\boxed{P \leftrightarrow Q \equiv Q \leftrightarrow P}$$

Note: if — then —

For only $(P \rightarrow Q) \Rightarrow$

Contrapositive $\sim Q \rightarrow \sim P$

$$\boxed{P \rightarrow Q \equiv \sim Q \rightarrow \sim P}$$

Converse $Q \rightarrow P$

Inverse $\sim P \rightarrow \sim Q$

e.g. ~~P~~

$P \rightarrow Q$: If a number is divisible by 9 then it is divisible by 3.

Contrapositive

$\sim Q \rightarrow \sim P$: If a number is not divisible by 3 then it is not divisible by 9.

Note:

Tautology ^(T) : Statement with T as all its truth values.

P	$\sim P$	$P \vee (\sim P)$
T	F	T
F	T	T

} Tautology

Fallacy / Contradiction ^(F)

Statement with 'F' as all its truth values.

P	$\sim P$	$P \wedge (\sim P)$
T	F	F
F	T	F

} Fallacy

Mathematical Reasoning

Set – I

(Solved Question)

Questions on Mathematical Reasoning→ Set-I Live Solution on YouTube

Which of the following sentences are statements?

- ① 15 is less than 8. (F) (Statement)
- ② 2 is a prime number. (T) (Statement)
- ③ Today is a sunny day. (Not a Statement)
- ④ Open the door. (Not a Statement)
- ⑤ All real numbers are complex numbers. (T) (Statement)
- ⑥ Mathematics is difficult. (Not a Statement)

Write the negation of the following statements.

- ⑦ 2 is a negative number.
→ 2 is not a negative number.

- ⑧ All triangles are not equilateral triangles.
→ All triangles are equilateral triangles

- ⑨ There exists a capital for every state in India.

Negation: There exists a state in India which does not ~~has~~ have a capital.

10 The negation of the statement "If I become a teacher, then I will open a school" is — AIEEE-2012

- (A) I will become a teacher and I will not open a school.
- (B) Either I will not become a teacher or I will not open a school.
- (C) Neither I will become a teacher nor I will open a school
- (D) I will not become a teacher or I will open a school.

Q.11 Consider the following statements:

P: Suman is brilliant.

Q: Suman is rich.

R: Suman is honest.

The negation of the statement

"Suman is brilliant and dishonest if and only if Suman is rich"

can be expressed as — AIEEE-2011

- (A) $\sim(P \wedge \sim R) \leftrightarrow Q$
- (B) $\sim P \wedge (Q \leftrightarrow \sim R)$
- (C) $\sim(Q \leftrightarrow (P \wedge \sim R))$
- (D) $\sim Q \leftrightarrow \sim P \wedge R$

Explanation:

$$\sim((P \wedge \sim R) \leftrightarrow Q)$$

Q.12 Consider the following statements:

P : Ram is Intelligent.

Q : Ram is Strong.

Then symbolic form of statement

"It is wrong that Ram is intelligent or strong" is

(A) $\sim P \vee Q$

(B) $\sim P \wedge \sim Q$

(C) $\sim P \vee \sim Q$

(D) $P \wedge \sim Q$

↓

$$\sim (P \vee Q)$$
$$\Rightarrow (\sim P) \wedge (\sim Q)$$

Q.13 Write the Contrapositive of inverse of converse of

$P \rightarrow \sim Q$.

~~$Q \rightarrow \sim P$~~ ~~$P \rightarrow \sim Q$~~

(A) $Q \rightarrow \sim P$

(B) $\sim P \rightarrow Q$

(C) $P \rightarrow \sim Q$

(D) $\sim Q \rightarrow P$

Write the Contrapositive, Converse & Inverse

Q.14 If x is a prime number, then x is odd. ($P \rightarrow Q$)

Converse: $Q \rightarrow P$ If x is odd then x is a prime number.

Inverse: $\sim P \rightarrow \sim Q$ If x is not a prime no., then x is not odd.

Contrapositive: $\sim Q \rightarrow \sim P$ If x is not odd then x is not a prime number.

Q.15

JEE Main
2017

The following statement

$$(P \rightarrow Q) \rightarrow [(\sim P \rightarrow Q) \rightarrow Q]$$

is $\rightarrow$

- (A) a fallacy (B) a tautology
(C) $\sim P \rightarrow Q$ (D) $P \rightarrow \sim Q$

P	Q	$P \rightarrow Q$	$\sim P \rightarrow Q$	$\sim P$	$\sim Q$	$(\sim P \rightarrow Q) \rightarrow Q$	$(P \rightarrow Q) \rightarrow [(\sim P \rightarrow Q) \rightarrow Q]$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	F	T	T
F	F	T	F	T	T	T	T

Tautology

Q.16 The Boolean Expression $\sim(P \vee Q) \vee (\sim P \vee Q)$ is equivalent to —JEE Main
2018

- (A) $\sim Q$ (B) $\sim P$ (C) P (D) Q

option (B)

P	Q	$\sim P$	$\sim Q$	$P \vee Q$	$\sim P \vee Q$	$\sim(P \vee Q)$	$\sim(P \vee Q) \vee (\sim P \vee Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	T	T	F	T
F	F	T	T	F	T	T	T

Question

Q.17 Which of the following is true?

- ~~x~~ (A) $P \wedge \sim P \equiv t$ ~~x~~ (B) $P \vee \sim P \equiv f$
~~x~~ (C) $P \rightarrow Q \equiv Q \rightarrow P$ ~~(D)~~ $P \rightarrow Q \equiv \underbrace{\sim Q \rightarrow \sim P}_{\text{Contrapositive}}$

Q.18 If $P \rightarrow (Q \vee R)$ is false, then the truth values of P, Q, R are, respectively —

(A) F, T, T
↓
 $F \rightarrow (T \vee T)$
 $F \rightarrow T$
 T
X

(B) T, T, F
↓
 $T \rightarrow (T \vee F)$
 $T \rightarrow T$
 T
X

~~(C)~~ T, F, F
↓
 $T \rightarrow (F \vee F)$
 $T \rightarrow F$
 F
✓

~~(D)~~ F, F, F
↓
 $F \rightarrow (F \vee F)$
 $F \rightarrow F$
 T

Mathematical Reasoning

Set – II

(Only Question)

(Solutions are after them)

Questions on Mathematical Reasoning

→ Set-II (For Students)

Which of the following sentences are statements? (Q1 to Q5)

- ① Sun rises in the south.
- ② Answer this question.
- ③ This question is easy.
- ④ where are you?
- ⑤ We should tell the truth

Write the negation of following statements.
(Q6 to Q9)

- ⑥ Every natural number is an integer.
- ⑦ For every real number x , x is less than $x+1$.
- ⑧ There exists a number which is equal to its square.
- ⑨ There exists a number x such that $1 < x < 2$.

Q.10 Negation of "Jaipur is in Rajasthan and Indore is in Madhya Pradesh" is

- (A) Jaipur is in Rajasthan and Indore is in Madhya Pradesh.
- (B) Jaipur is in Rajasthan or Indore is not in Madhya Pradesh.
- (C) Jaipur is not in Rajasthan or Indore is not in Madhya Pradesh.
- (D) Jaipur is not in Rajasthan and Indore is not in Madhya Pradesh.

Q.11 Consider the following statements:

p: It rains today

q: I go to school

r: I shall meet my friends

s: I shall go for a movie.

Then which of the following propositions represents "If it does not rain or if I do not go to school, then I shall meet my friends and go for a movie"?

- (A) $\sim(p \wedge q) \rightarrow (r \wedge s)$
- (B) $\sim(p \wedge \sim q) \rightarrow (r \wedge s)$
- (C) $\sim(p \wedge q) \rightarrow (r \vee s)$
- (D) $\sim(p \vee q) \rightarrow (r \wedge s)$

Q.12 If P : "Shyam is tall"
and q : "Shyam is intelligent",
then the statement $(\sim p \vee q)$ is

- (A) Shyam is not tall or he is intelligent.
- (B) Shyam is tall or he is intelligent.
- (C) ~~Shyam~~ is not tall and he is intelligent.
- (D) Shyam is not tall then he is intelligent.

Q.13 Contrapositive of converse
of ' $P \rightarrow q$ ' is

- (A) $P \rightarrow q$ (B) $q \rightarrow P$
- (C) $\sim P \rightarrow \sim q$ (D) $\sim q \rightarrow \sim p$

Q.14

Write the contrapositive, converse
& Inverse of the following
Statement -

" x is Even number implies that
 x is divisible by 4"

Q.15 The statement $\sim(p \leftrightarrow \sim q)$ is

- (A) equivalent to $p \leftrightarrow q$
- (B) equivalent to $\sim p \leftrightarrow q$
- (C) a tautology
- (D) a fallacy.

Q.16 The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to

- (A) $s \wedge \sim r$
- (B) $s \wedge (r \wedge \sim s)$
- (C) $s \vee (r \vee \sim s)$
- (D) $s \wedge r$

Q.17 The boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to

- (A) $p \wedge q$
- (B) $p \vee q$
- (C) $p \vee \sim q$
- (D) $\sim p \wedge q$

Q.18 $\sim((\sim(\sim p)) \wedge q)$ is equal to

- (A) $\sim p \wedge q$
- (B) $\sim p \vee \sim q$
- (C) $p \wedge \sim q$
- (D) $\sim p \wedge \sim q$

Mathematical Reasoning

Set – II

(Solutions)

Solution of Set-II

pdf on $\rightarrow$ www.toppersvillage.com

- ① Statement
- ② Not Statement
- ③ Not Statement
- ④ Not Statement
- ⑤ Not Statement
- ⑥ Every natural number is not an integer.
- ⑦ There exists a real number x such that x is not less than $x+1$.

⑧ There does not exist a number which is equal to its square.

⑨ There does not exist a number x such that ~~$1 < x < 2$~~
 $1 < x < 2$.

⑩ Correct option = (c)

Let P : Jaipur is in Rajasthan.

Q : Indore is in Madhya Pradesh.

So we want negation of ' P ' & ' Q '

$\Rightarrow$ we want $\sim(P \wedge Q)$

$$= (\sim P) \vee (\sim Q)$$

$$\boxed{\because \sim(P \wedge Q) \equiv (\sim P) \vee (\sim Q)}$$

See theory $\Rightarrow$ 'and' convert into 'or'

⑪ After reading the given statement, we firstly get

$$(\sim P) \vee (\sim Q) \rightarrow (R \wedge S)$$

$$= [\sim(P \wedge Q)] \rightarrow (R \wedge S)$$

$$= \sim(P \wedge Q) \rightarrow (R \wedge S)$$

$\therefore$ Correct option = (A)

⑫

$\sim P$: Shyam is not tall

~~$\vee$ or~~

Q : Shyam is intelligent

$\therefore (\sim P \vee Q)$: Shyam is not tall or Shyam is intelligent

Correct option = (A)

Q.13

Given statement: $P \rightarrow Q$

Converse of it: $Q \rightarrow P$

Contrapositive of converse of it: $\sim P \rightarrow \sim Q$

$\therefore$ Correct option = (C)

Q.14

Let P : x is even number.

Q : x is divisible by 4.

— implies that — $\equiv$ If — then —

$\therefore P \rightarrow Q$ is given

Contrapositive: $(\sim Q \rightarrow \sim P)$: If x is not divisible by 4 then x is not even number.

Converse: $(Q \rightarrow P)$: If x is divisible by 4 then x is even number.

Inverse: $(\sim P \rightarrow \sim Q)$: If x is not even number then x is not divisible by 4.

Q.15 Truth table

P	Q	$\sim P$	$\sim Q$	option(A) $P \leftrightarrow Q$	option(B) $\sim P \leftrightarrow Q$	Question $P \leftrightarrow \sim Q$	$\sim(P \leftrightarrow \sim Q)$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
F	F	T	T	T	F	F	T

$\therefore$ ~~Corrected~~ option = (A) ✓

Q.16 Truth table

S	γ	$\sim S$	$\sim \gamma$	$\sim \gamma \wedge S$	Negation of this $\sim(\sim S \vee (\sim \gamma \wedge S))$	option(A) $S \wedge \sim \gamma$	option(B) $S \wedge (\gamma \wedge \sim S)$	option(C) $S \vee (\gamma \vee \sim S)$	option(D) ✓ $S \wedge \gamma$
T	T	F	F	F	F (T)	F	F	T	T
T	F	F	T	T	T (F)	T	F	T	F
F	T	T	F	F	T (F)	F	F	T	F
F	F	T	T	F	T (F)	F	F	T	F

we get this
(so these are the truth values
of question)

we got the
match
 $\therefore$ Correct option = (D)

Q.17

OPTION D

Question

OPTION A

OPTION B

OPTION C

P	q	$\sim P$	$\sim q$	$P \wedge \sim q$	$\sim P \wedge q$	$(P \wedge \sim q) \vee q \vee (\sim P \wedge q)$	$P \wedge q$	$P \vee q$	$P \vee \sim q$
T	T	F	F	F	F	T	T	T	
T	F	F	T	T	F	T	F	T	
F	T	T	F	F	T	T	F	T	
F	F	T	T	F	F	F	F	F	

So the correct option = (B)

Q.18

$\therefore (\sim(\sim P)) \equiv P \quad \therefore \sim[(\sim(\sim P)) \wedge q] \leftarrow \text{our question}$

$$\equiv \sim[P \wedge q]$$

$$\equiv (\sim P) \vee (\sim q) \leftarrow \text{option} = (B)$$

(See theory)

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>